CS-2209A

Assignment 2

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1. P(x) = pens

Q(x) = pencils

* 1. ∀x (P(x)∨Q(x))
  2. ∃x (¬(P(x))→(Q(x)
  3. ∃x (¬P(x)∧¬Q(x))
  4. ¬∀x(P(x)∧Q(x))



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| 1 | ∀z((∃yP(x,y)→∀xQ(x))→R(z)) | Question |
| 2 | ∀z(¬(∃yP(x,y)→∀xQ(x))∨R(z)) | Conditional Identity 1 |
| 3 | ∀z(¬(¬∃yP(x,y)∨∀xQ(x))∨R(z)) | Conditional Identity 2 |
| 4 | ∀z((¬¬∃yP(x,y)∧¬∀xQ(x))∨R(z)) | DeMorgans Law 3 |
| 5 | ∀z((∃yP(x,y)∧¬∀xQ(x))∨R(z)) | Double Negation 4 |
| 6 | ∀z((∃yP(x,y)∧∃x¬Q(x))∨R(z)) | DeMorgans Law 5 |
| 7 | ∀z∃y∃x((P(x,y)∧¬Q(x))∨R(z)) | Shifting Quantifiers 6 |
| 8 | ∀z∃y∃x((R(z)∨P(x,y))∧(R(z)∨¬Q(x))) | Distributive Law 7 |



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| 1 | ((A(y)→∀x1B(x1))→∃x2:C(x) | Question |
| 2 | ¬((A(y)→∀x1B(x1))∨∃x2 C(x2) | Conditionally Identity 1 |
| 3 | ¬(¬(A(y)∨∀x1B(x1))∨∃x2 C(x2) | Conditional Identity 2 |
| 4 | (¬¬A(y)∧¬∀x1B(x1))∨∃x2 C(x2) | DeMorgans Law 3 |
| 5 | (A(y)∧¬∀x1B(x))∨∃x2 C(x2) | Double Negation Law 4 |
| 6 | (A(y)∧∃x1¬B(x1))∨∃x2 C(x2) | DeMorgans Law 5 |
| 7 | (∃x2C(x2)∨A(y))∧ (∃x2C(x2)∨ ∃x1¬B(x1)) | Distribution Law 6 |
| 8 | ∃x2∃x1 (C(x2)∨A(y))∧ (C(x2)∨¬B(x1)) | Shifting Quantifiers 7 |



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| 1 | ∃x1(P(x1)→∀yQ(x1,y))→∀x2B(x2) | Question |
| 2 | ∃x1¬(P(x1)→∀yQ(x1,y))∨∀x2B(x2) | Conditional Identity 1 |
| 3 | ∃x1¬(¬P(x1)∨∀yQ(x1,y))∨∀x2B(x2) | Conditional Identity 2 |
| 4 | ∃x1¬(¬P(x1)∨∀yQ(x1,y))∨∀x2B(x2) | DeMorgans Law 3 |
| 5 | ∃x1(¬¬P(x1)∧∃y¬Q(x1,y))∨∀x2B(x2) | DeMorgans Law 4 |
| 6 | ∃x1(P(x1)∧¬∃y¬Q(x1,y))∨∀x2B(x2) | Double Negation Law 5 |
| 7 | ((∀x2B(x2)∨∃x1P(x1))∧ ((∀x2B(x2)∨∃y¬Q(x1,y)) | Distributive Law 6 |
| 8 | ∀x2∃x1∃y((B(x2)∨P(x1))∧ ((B(x2)∨¬Q(x1,y)) | Shifting Quantifiers 7 |



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| 1 | ∀x(P(x)→Q(x)) | Hypothesis |
| 2 | ∀x(Q(x)→R(x)) | Hypothesis |
| 3 | ∀x(P(x)→R(x)) | Hypothetical Syllogism 1,2 |



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| 1 | ∃x(P(x)∧R(x)) | Hypothesis |
| 2 | (c is a particular element) ∧(P(c)∧R(c)) | Existential instantiation 1 |
| 3 | (c is a particular element) | Simplification 2 |
| 4 | (P(c)∧R(c))∧(c is a particular element) | Commutative law 2 |
| 5 | P(c)∧R(c) | Simplification 4 |
| 6 | ∀x(P(x)→(Q(x)∧S(x)) | Hypothesis |
| 7 | P(c)→(Q(c)∧S(c)) | Universal instantiation 4,5 |
| 8 | ¬P(c)∨(Q(c)∧S(c)) | Conditional Identity 7 |
| 9 | ¬P(c)∨(S(c)∧Q(c)) | Commutative Law 8 |
| 10 | ¬P(c)∨S(c) | Simplification 9 |
| 11 | S(c)∨¬P(c) | Commutative Law 10 |
| 12 | (S(c)∨¬P(c))∧(P(c)∧R(c)) | Conjunction 5,11 |
| 13 | S(c)∧F∧R(c) | Complement Law 12 |
| 14 | S(c)∧R(c) | Domination Law |
| 15 | R(c)∧S(c) | Commutative Law 14 |
| 16 | (c is a particular element)∧(R(c)∧S(c)) | Conjunction 3,15 |
| 17 | ∃x(R(x)∧S(c)) | Existential Generalization 16 |



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| 1 | ∀x(P(x)∨Q(x)) | Hypothesis |
| 2 | ∀x(¬Q(x)∨S(x)) | Hypothesis |
| 3 | ∀x(R(x)→¬S(x)) | Hypothesis |
| 4 | ∃x(¬P(x)) | Hypothesis |
| 5 | (c is a particular element)∧(¬P(c)) | Existential Instantiation 4 |
| 6 | (¬P(c))∧( c is a particular element) | Commutative Law 5 |
| 7 | C is a particular element | Simplification 5 |
| 8 | P(C)∨Q(c) | Universal Instantiation 1, 7 |
| 9 | ¬Q(c)∨S(c) | Universal Instantiation 2,7 |
| 10 | R(c)→¬S(c) | Universal Instantiation 3,7 |
| 11 | ¬P(c) | Simplification 6 |
| 12 | Q(c)∨P(c) | Commutative Law 8 |
| 13 | P(c)∨S(c) | Resolution 9,12 |
| 14 | S(c) | Disjunctive Syllogism 11,13 |
| 15 | ¬R(c) | Modus Tollens 10,14 |
| 16 | (c is a particular element) ∧ ¬R(x) | Conjunction 7,16 |
| 17 | ∃x(¬R(x)) | Existential Generalization 16 |

1. 1. P(x): some pizza store open in London

H(x): there is a party

s: there is a party at Sally’s house

d: there is a party at Dan’s house

∃xP(x)→H(d)

¬H(d)∧¬H(s)

∀x¬P(x)

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| 1 | ∃xP(x)→H(d) | Hypothesis |
| 2 | ¬H(d)∧¬H(s) | Hypothesis |
| 3 | ¬H(d) | Simplification 2 |
| 4 | ¬∃xP(x) | Modus Tollens 1,3 |
| 5 | ∀x¬P(x) | De Morgans Law 4 |

Therefore the statement is valid

* 1. P: Darryl is happy

Q: it is raining outside

R: pigs have wings

p∨q

¬q∨r

R

¬p

To prove validity, we check

(p∨q)∧(¬q∨r)∧(r)→¬p: C

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| P | Q | R | ¬p | ¬Q | p∨q | ¬q∨r | (p∨q)∧(¬q∨r) | (p∨q)∧(¬q∨r)∧(r) | C |
| T | T | T | F | F | T | T | T | T | F |
| T | T | F | F | F | T | F | F | F | T |
| T | F | T | F | T | T | T | T | T | F |
| T | F | F | F | T | T | T | T | F | T |
| F | T | T | T | F | T | T | T | T | T |
| F | T | F | T | F | T | F | F | F | T |
| F | F | T | T | T | F | T | F | F | T |
| F | F | F | T | T | F | T | F | F | T |

From the truth table we can see that the argument is not valid as not all truth values of C are true

1. 1. 2.5.8.
      1. ∀x∀y W(x)∧O(y)→N(x,y)
      2. ∃x∀y O(x)→¬N(x,y)
      3. ∀x∃y N(x,y)
      4. ∃x∀y N(x,y)
      5. ∀x(∃y N(x,y))
      6. ∃x N(x,sam)
   2. 2.4.2
      1. E) ∀x∃y Q(x,y) False, when x=1 there is no value of y that is true
      2. F) ∀x∃y P(x,y) True, for all x there is at least one y value that is true
      3. G) ∀x∀y P(x,y), false P(2,1), P(2,2),P(3,3) are all false
      4. I) ∀x∀y ¬S(x,y), true all values that exist are true
2. 1. Theorem: , x,y are positive integers

Proof:

x-1 and x are two consecutive positive integers

Let p divide x-1 and x where p>= 1 is a positive integer

Then x -1 = pt1

X=pt2 and t2>t1, such that t2 and t1 are positive integers

So x-(x-1)=pt2-pt1

1= p(t2-t1)

And p is positive in positive integers t2-t1>0 is a positive integer

So p=1, t2-t1=1

So if px and p(x-1) then p =1

So x, x-1 = 1

Hence 1/y and x-1/x are rational numbers with 1/y ae co-prime and x-1, x are coprime

So that 1/y=x-1/x

1= x-1, y=x

Y=x=2

* 1. Theorem: For any positive integer a, if a3 is an even number then a is an even number.

Proof: If a is an odd number, then a3 is odd

Then a = 2m+1 for some integer

And a3 = (2m+1)3

= (2m)3 + 13 + 3(2m)3 +3\*2m\*12

= 8m3+12m2+6m +1

= 2(4m3 +12m2+3m)+1

Let n=(4m3 +12m2+3m)

= 2n+1

So a3 is an odd number hence if a is not even then a3 is not even or if a3 is even then a is an even number

* 1. Theorem: is not a rational number

Proof: =a/b (we assume that a and b have no common factors and are therefore irreducible)

2=a3/b3

2b3=a3

Let b3 =n

2n=always even

=>a3 must be even

=a\*a\*a is even

= even\*even\*even => even

* A is even =>a=2k

2b3=(2k)3

2b3=8k3

B3=4k3

* B3 must be even
  + =4k3 must be even
  + B is even

If a and b are even then they both have common factors of 2 and therefore the original assumption of the a/b being irreducible is not true, this is a contradiction of the original statement therefore is not a rational number

* 1. Theorem: if (a, b >0) then a/b if there exists an integer k such that b=ka

If a>1 then a does not divide a2+a+1

Proof:

a = a>1

b=a/a2+a+1

a→b

contrapositive ¬b→¬a

(a/a2+a+1)→1≥a

1≥a

Ka = a2+a+1 where k is an integer

Therefore, the argument is invalid and by contrapositive the original theorem is true